RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2024
SECOND YEAR (BATCH 2022-24)
MATHEMATICS
Paper : CC16

Date : 09/05/2024
Time : 11.00 am – 1.00 pm
Full Marks : 50

[All the notations have their usual meaning.]

Group:- A

Unit : I

Answer **any two** questions: [2×10]

1. a) Find Fourier sine and cosine transform of $F(x)=x$.

   b) Find $L^{-1}\left\{\frac{5}{s^2}+\left(\frac{\sqrt{s}-1}{s}\right)^2-\frac{7}{3s+2}\right\}$. [6+4]

2. a) Solve the system of equations $Dx(t)+5x(t)-2y(t)=t; Dy(t)+2x(t)+y(t)=0$, where $D\equiv\frac{d}{dt}, x(0)=0$ and $y(0)=2$ by using Laplace transformation.

   b) If $f(s)$ and $g(s)$ are the Fourier transforms (complex) of $F(x)$ and $G(x)$ respectively then show that $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(s)\overline{g(s)}ds=\int_{-\infty}^{\infty} F(x)\overline{G(x)}dx$. [7+3]

3. a) Solve (using Fourier sine transform) $u_t=u_{xx},\ x>0, t>0$ subject to the condition $u(0,t)=0,\ u(x,0)=\begin{cases}1, & 0<x<1\\ 0, & x\ge 1\end{cases},\ u(x,t)$ is bounded; u_x and u tends to 0 as $x\to\infty$.

   b) Use Laplace transform to find the value of the integral $\int_0^{\infty} e^{-x^2}dx$. [7+3]

Unit : II

Answer **any two** questions: [2×7.5]

4. Solve the integral equation $y(x)=\lambda\int_0^{2\pi}\sin(x+t)\,y(t)\,dt$.

5. Determine the iterated kernels of $y(x)=f(x)+\lambda\int_0^{\pi}e^x\cos t\,y(t)\,dt$. (find upto 4^{th} order and then generalize)

6. Find the Neumann series solution of the integral equation

   $$y=1+x+\lambda\int_0^x (x-t)\,y(t)\,dt.$$

   Hence show that e^x is the exact solution of the given integral equation for $\lambda=1$.

Group:- B

Answer **any two** questions: [2×7.5]

7. Find the shortest distance between the point $P_1(1,0)$ and the ellipse $4x^2+9y^2=36$.

8. Find the extremal for the functional $I[y(x)]=\int_0^{\pi/2}\left(y^2-(y')^2-2y\sin x\right)dx$

   subject to the conditions : $y(0)=0$ and $y\left(\frac{\pi}{2}\right)=0$.

9. Deduce Euler-Poisson equation. $\Big($consider the functional as $I[y(x)]=\int_a^b f(x,y,y',y'')\Big)$.

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